Munich Center for Machine Learning
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Confidence in Causal Inference under Structure Uncertainty

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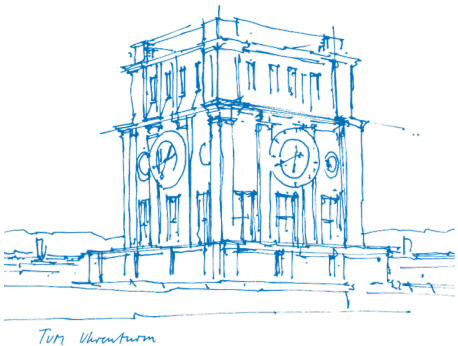


Starting point

- **Research question:** What is the total causal effect of X_i on X_j? Confidence?
- **Given:** Observational data in form of n samples of $(X_1, ..., X_d)$.
- **Problem:** Causal structure unknown.

Starting point

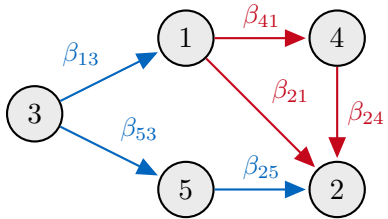
- **Research question:** What is the total causal effect of X_i on X_j? Confidence?
- **Given:** Observational data in form of n samples of $(X_1, ..., X_d)$.
- **Problem:** Causal structure unknown.
- Naive two-step approach?
  (1) Learn causal structure.
  (2) Calculate confidence intervals for causal effects in inferred model.

Setup

Underlying Linear SEM with equal error variances

- **Example:** Target effect $\mathcal{C}(1 \to 2) := \frac{\mathrm{d}}{\mathrm{d}x_1}\mathbb{E}[X_2|\ \mathrm{do}(X_1 = x_1)] = \beta_{21} + \beta_{41}\beta_{24}$.

$$
\begin{aligned}
X_1 &= \beta_{13}X_3 + \varepsilon_1 \\
X_2 &= \beta_{21}X_1 + \beta_{24}X_4 + \beta_{25}X_5 + \varepsilon_2 \\
X_3 &= \varepsilon_3 \\
X_4 &= \beta_{41}X_1 + \varepsilon_4 \\
X_5 &= \beta_{25}X_3 + \varepsilon_5
\end{aligned}
$$



where $\varepsilon_j \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma^2)$

Setup

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- **Goal:** Construct suitable **tests for all possible effects.**

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- **Difficulty:** Each Hypothesis of fixed effect is **union of single hypotheses** over all DAGs on d nodes.

$$\mathsf{H}_0^{(\psi)} := \bigcup_{G \in \mathcal{G}(d)} \mathsf{H}_0^{(\psi)}(G)$$

Main Result

- **Main steps**:

  (1) Intersection union test.

  (2) Stochastic upper bound by relaxing alternative.

  (3) LRT with conservative critical values from upper bound.

Main Result

- **Main steps**:
  - (1) Intersection union test.
  - (2) Stochastic upper bound by relaxing alternative.
  - (3) LRT with conservative critical values from upper bound.
- **Result:** Asymptotic $(1-\alpha)$-confidence set for causal effect $\mathcal{C}(i \to j)$ is

$$\{\psi \in \mathbb{R} : \min_{G \in \mathcal{G}(d) : i <_G j} \lambda_n^{(\psi)}(G) \leq \chi^2_{d,1-\alpha}\} \cup \{0 : \min_{G \in \mathcal{G}(d) : j <_G i} \lambda_n^{(0)}(G) \leq \chi^2_{d-1,1-\alpha}\}$$

Conclusion

- Confidence regions for total causal effects capturing both types of uncertainty: numerical size of effects and causal structure.
- Branch and bound type search algorithm through causal orderings. Feasible up to 12 involved variables (already more than 10^{26} structures).
- Conceptual idea of leveraging test inversions of joint tests for causal structure and effect size generalizable to other modeling assumptions.