Research group Mathematical Statistics
TUM School of Computation, Information and Technology
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Confidence in Causal Discovery with Linear Causal Models

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First results in:
Strieder, D., Freidling, T., Haffner, S., Drton, M. Confidence in causal discovery with linear causal models. PMLR 161:1217-1226 (2021).

17. Doktorand:innentreffen der Stochastik
University of Klagenfurt
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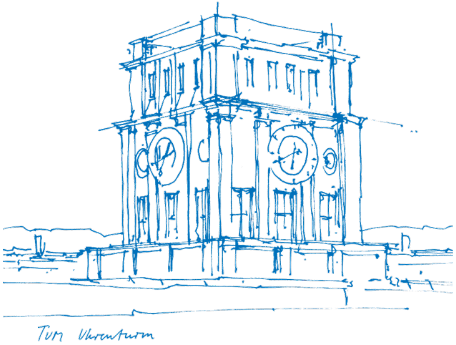

Starting point

- Given: Observational data in form of n samples of $(X_1, ..., X_d)$.

- Research question: What is the (total) causal effect of X_1 on X_2? Confidence?

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- Naive two-step approach?
  (1) Learn causal structure.
  (2) Calculate confidence intervals for causal effects in inferred model.

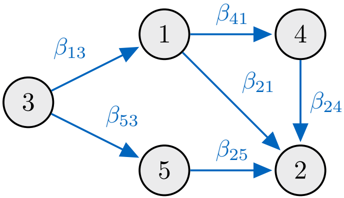
Setup
Model assumptions that ensure identifiability

- **Linear** structural equation model with **Gaussian errors** with **equal variances**.

LSEM

$$X_j = \sum_{k \neq j} \beta_{jk} X_k + \epsilon_j, \qquad \epsilon_j = N(0, \sigma^2), \qquad j = 1, ..., d.$$

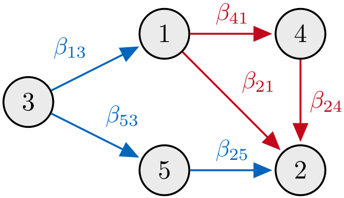
- Represented by directed acyclic graph.



Setup

- Target: **Total causal effect**

$$\mathcal{C}(1 \to 2) := \frac{\mathsf{d}}{\mathsf{d}x_1}\mathbb{E}[X_2|\,\mathsf{do}(X_1 = x_1)] = \Sigma_{12|pa(1)}/\Sigma_{11|pa(1)}\mathbb{1}(1 < 2)$$

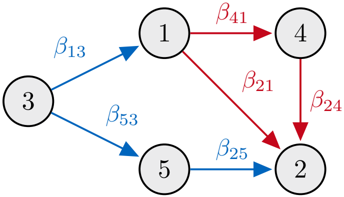


Setup

- Idea: Duality between statistical hypothesis test and confidence regions.

Tests for $H_0 : \mathcal{C}(1 \to 2) = \psi$ → Confidence interval for $\mathcal{C}(1 \to 2)$

- Goal: construct suitable **tests for all possible hypothesized causal effects!**



Hypothesis $\mathcal{C}(1 \to 2) = \psi$

Case $\psi \neq 0$

$$\cup_{1<2}\left\{\Sigma \in \text{PD(d)} : \exists\sigma^2 \text{ such that } \begin{cases} \psi & = \Sigma_{12|pa(1)}/\sigma^2 \\ \sigma^2 & = \Sigma_{jj|pa(j)} \quad \text{for all } j \in 1, ..., d \end{cases}\right\}$$

Case $\psi = 0$

$$\cup_{1<2} \begin{cases} 0 & = \Sigma_{12|pa(1)}/\sigma^2 \\ \sigma^2 & = \Sigma_{jj|pa(j)} \quad \text{for all } j \in 1, ..., d \end{cases} \bigcup \cup_{2<1} \left\{\sigma^2 \quad = \Sigma_{jj|pa(j)} \quad \text{for all } j \in 1, ..., d\right.$$

Next Steps?

- Intersection union test.
- Relax alternative, single Hypothesis is submanifold of pd cone.
- Theory of constrained likelihood ratio tests.
- Maximizing Gaussian likelihood under polynomial constraint.

Bivariate Example

Theorem. *Let* $\alpha \in (0,1)$ *and define*

$$K_a := 2\,\hat{\Sigma}_{aa}\,\det(\hat{\Sigma})^{1/2}\exp\left(\frac{1}{2n}\chi^2_{3-a,1-\alpha}\right) - \hat{\Sigma}_{aa}^2 - \det(\hat{\Sigma}), \quad a = 1,2.$$

Then an asymptotic $(1-\alpha)$ *confidence set for the causal effect* $\mathcal{C}(1 \to 2)$ *is given by:*
(i) If $K_1 \geq 0$*, then the nonzero elements of the confidence set are the nonzero elements of the interval* $[L, U]$ *with*

$$L := \frac{\hat{\Sigma}^0_{12} - \sqrt{K_1}}{\hat{\Sigma}^0_{11}}, \quad U := \frac{\hat{\Sigma}^0_{12} + \sqrt{K_1}}{\hat{\Sigma}^0_{11}}.$$

(ii) Additionally the confidence set contains zero if and only if $K_2 \geq 0$*.*

Simulations